



**Open Question 1. “The Model That Could Not Cross the Border”:
exporting frontier AI**

Frontier AI systems are dual-use technologies: they may help firms improve productivity, but at the same time they may help foreign users discover and exploit vulnerabilities or attempt to replicate some of the model's capabilities or extract sensitive information from it.

Country A's firm AlphaAI owns the best AI model and sells monitored access to entities in Country B (one unit = one standardized access contract during the policy period).

Country B's inverse demand is $P = 100 - Q$. AlphaAI's marginal cost is 20 and it is a foreign monopolist in Country B.

The government of Country A believes exports create an expected national-security external cost $E(Q) = 8 \cdot Q + Q^2/2$ (where $8 \cdot Q$ captures an ordinary misuse risk, while $Q^2/2$ is due to risks rising disproportionately with scale, e.g. jailbreaks).

Country A defines welfare as: AlphaAI profit + government license revenue – external cost.

(a) (8 points) Suppose exports are unregulated. With unregulated exports, find AlphaAI's profit, the external cost, and Country A's welfare.

Is a complete export ban better for Country A compared to unregulated exports? Explain.

(b) (8 points) Instead of a ban, suppose Country A requires AlphaAI to buy one export license for every unit of access sold abroad. Find the per-unit license fee that maximizes Country A's welfare. Compare this policy with both unregulated exports and a complete ban on profit, government revenue, welfare, and the external cost.

(c) (4 points) Instead, Country A imposes an export quota equal to the quantity found in part (b). Consider two methods of allocating the quota rights:

- (i) the rights are given to AlphaAI for free;
- (ii) AlphaAI pays the government, for each unit of quota used, the per-unit fee found in part (b).

Under each method, calculate AlphaAI's profit, government revenue, Country A's welfare, and the external cost. Identify which parameters differ between the two methods and which remain unchanged.

(d) (5 points) Alternatively AlphaAI sells a copy of the model weights outright to a foreign firm, for a one-time payment of 900. There would be no license revenue afterwards. Country A's government estimates that this sale would create an external cost of 700.

Would AlphaAI prefer this to the licensing like in (b)? Would Country A prefer it? Explain.

(e) (5 points) After Country A restricts access, Country B builds its own substitute AI model. Give one potential benefit and two potential costs/risks of this response for Country A.

Open Question 2. Mutual funds, fees, and closet indexing

A mutual fund pools money from many investors and invests it on their behalf. Some mutual funds are passively managed: they simply track a benchmark, such as a stock index. Other mutual funds are actively managed: the fund manager tries to beat the benchmark by choosing investments.

Mutual fund managers are often paid a management fee equal to a percentage of the fund's assets under management, or AUM.

Suppose investors are risk neutral. They care only about expected return, not about risk. Each investor can always put money into a passive benchmark fund that earns zero excess return. Therefore, an active mutual fund can attract money only if investors expect its net excess return, after management fees, to be at least zero. If expected net excess return is positive, investors are willing to supply unlimited funds to the mutual fund. If expected net excess return is negative, no investor is willing to invest. In equilibrium, if the fund receives any money, its expected net excess return must be exactly zero.

Suppose a fund manager has skill, but there are decreasing returns to scale. If the manager actively manages x billion dollars, the fund generates total gross excess return: $G(x) = 0.08 \cdot x - 0.005 \cdot x^2$ (billions per period).

(a) (12 points) Suppose the manager freely sets a fee rate and actively invests all money.

(i) If the fund has x billion dollars under management, what is the maximum fee rate $f(x)$ that investors are willing to pay?

(ii) Which x maximises the manager's fee revenue $f(x) \cdot x$?

(iii) What fee rate does the manager charge, and how much money does the manager earn in one period?

(b) (12 points) Now suppose regulation or market convention fixes the fee at 2 percent of AUM. The manager can actively manage only x billion dollars and put the rest of the fund into the passive benchmark. This is called closet indexing: the fund charges active-management fees on all AUM, while some money is invested passively. Find:

(i) If the manager actively manages x billion dollars, what is the maximum total AUM $Q(x)$ that the fund can attract?

- (ii) Which x maximises the manager's fee revenue?
 - (iii) How much total AUM does the fund attract?
 - (iv) How much money is invested passively?
 - (v) How much does the manager earn in fees?
- (c)** (6 points) Suppose a fund had very high excess returns during the last period, so a rational investor infers the manager is highly skilled. Should the investor expect positive net excess returns next period? Explain.

Open Question 3. Designed here, assembled there

A finished chip requires one design blueprint (D) and one manufactured component (C): $\text{chips} = \min(D, C)$. Each worker performs one activity, and may divide working time between activities. Per period:

- an engineer produces 2 blueprints or 1 component;
- a machinist produces 1 blueprint or 3 components.

There are two countries in the world: Northia and Fabrica. Northia has 60 engineers and 20 machinists. Fabrica has 20 engineers and 60 machinists. Countries consume only finished chips, assembled domestically.

(a) (6 points) Each country lives in autarky. Find the maximum number of chips each country can produce. Explain why the opportunity cost of blueprints in terms of components is increasing in each country, even though every individual worker has constant productivity.

(b) (7 points) Blueprints and components can now be traded freely, but labor cannot migrate. Find the maximum world chip output and describe who does what. Is each country fully specialized? Explain why or why not.

(c) (9 points) Northia's parliament, worried that "we design brilliantly but have lost the ability to make things," caps component imports, forcing some engineers into manufacturing. The government announces: "The policy created new jobs." Evaluate three claims without detailed calculation:

- (i) the claim that employment was increased;
- (ii) a Fabrica newspaper's claim that Northia's quota will *help* Fabrica's design sector;
- (iii) Northia's claim that the quota is insurance: "if trade is ever cut off completely, we will be better prepared." This is not true *within this model*, but in reality, this argument can make sense. What real-world feature, absent from the model, makes this true?

(d) (8 points) The production function $\text{chips} = \min(D, C)$ assumes one blueprint is "used up" per chip. In the real semiconductor industry, a single design is copied onto millions of chips at almost zero extra cost, while manufacturing capacity

requires enormous investment. Suppose we replaced the assumption with: *one blueprint is enough for any number of components, but designing a blueprint requires 10 engineers*. Without calculations, explain:

- (i) how the world's optimal allocation of workers between design and manufacturing would change;
- (ii) why, under this technology, a country specializing in design might capture a share of world income far larger than its share of world labor.

Open Question 4. Utility maximization, social preferences, and public goods

Anil has won £100 in the lottery and is deciding how to split it between himself (x , the amount he keeps) and his friend Bala (y , the amount he gives). Suppose Anil has the utility function $u(x, y) = x - \delta \cdot (y - x)^2$, where $\delta \geq 0$ represents Anil's degree of inequality aversion.

- (a)** (6 points) Suppose you observe Anil giving £37.5 (out of £100) to Bala and keeping the rest for himself. What value of δ is consistent with this behaviour?
- (b)** (6 points) How does the amount Anil gives to Bala vary with δ ? Refer to Anil's utility function when explaining your answer. (You can give a mathematical and/or an intuitive explanation.)
- (c)** (6 points) Now suppose that Anil becomes purely selfish and plays a one-round game with Bala, Cindy and David (4 people total).
 - Each player is given £20 and chooses a contribution to a common pot
 - At the end of the round, every player receives $k = 0.2$ times the total amount in the pot plus the money they chose not to contribute.
 - If a player contributes less than some expected amount C , they are punished: an amount P deducted from their money at the end of the round.

Suppose $P = £6$. What is the largest minimum contribution that Anil would be willing to make (that is, find maximum C that he will agree to contribute)? Assume that he meets the requirement when indifferent.

(d) (6 points) If the share of the total amount in the pot changes to $k = 0.5$, what is the largest minimum contribution C Anil would be willing to contribute if the punishment is $P = £5$?

(e) (6 points) Before the round, the four players jointly agree on a single punishment size P and a single required minimum contribution C that will apply to all of them. Once play begins each player is again purely selfish. Anticipating this behaviour, the group picks P and C to maximise the combined end-of-round wealth of all four players. If indifferent between different levels of P , they choose the smallest. Determine the P and C the group should agree on when (i) $k = 0.2$ and (ii) $k = 0.5$, and give the resulting combined wealth in each case.

OQ 5. AI data centres: Earth or orbit? (30 points)

Firm SpaceY must decide whether to locate an AI training facility on Earth or in low-Earth orbit. Consider sunlight in orbit is free and continuous, but every piece of orbital equipment must first be launched from Earth.

Assume the orbital facility is technically feasible and ignore every private cost not stated explicitly: computing hardware, cooling and heat rejection, communications, radiation shielding, maintenance, insurance, and end-of-life disposal. All quantities and prices below are hypothetical.

(a) (6 points) In choosing a location, is an AI training data centre more like a shopping mall or an aluminium smelter facility? Name the location principle at work, and state which location factors matter most and which barely matter for such a centre.

(b) (9 points) Electricity on Earth costs \$0.05/kWh. A centre draws power continuously, operating 8,000 hours per year for 5 years. Ignore discounting unless explicitly asked. On Earth the firm buys grid power. In orbit power is free, but supplying 1 kW of continuous power requires launching 10 kg of equipment, at a current launch price of \$1,500/kg.

(i) For a 1 kW load, compute the lifetime Earth electricity bill and the orbital launch cost.

(ii) At what launch price per kg does orbit just break even? By what factor must today's launch price fall to reach it?

(iii) Ignoring discounting was assumed above. Does that assumption favour Earth or orbit, and why?

(c) (8 points) Today a firm pays to launch, but pays nothing else. Identify two costs an orbital centre imposes on parties other than the firm and its customers, and state who bears each. Then name precisely the economic problem associated with these costs. Choose one of the two suggested costs and propose a corrective instrument and the single obstacle to enforcing it.

(d) (7 points) Some regions have abundant electricity that is effectively stuck (stranded) — remote hydro with no transmission line, associated gas which is burnt off — so its local value is near zero. Electricity is costly to transport and to store; a trained model is a small file (weights). Explain in what sense building an AI training centre at such a site lets a firm "export electricity" that otherwise would be difficult to sell.